

Индивидуальное домашнее задание по теме:
Первообразная. Интеграл.

1. Вычислите определенный интеграл:

1. а) $\int_{-1}^1 (x^2 + 1) dx$

б) $\int_{-2}^{-1} \frac{dx}{x^4}$

в) $\int_0^{\frac{\pi}{2}} 8 \sin \frac{x}{2} \cos \frac{x}{2} dx$

2. а) $\int_{-2}^1 x(x+3)(2x-1) dx$

б) $\int_{-3}^{-1} \frac{dx}{x^6}$

в) $\int_0^{\frac{\pi}{8}} (\sin^2 2x - \cos^2 2x) dx$

3. а) $\int_{-1}^0 (3x^2 + 1) dx$

б) $\int_{-5}^{-3} \frac{dx}{(x+2)^2}$

в) $\int_0^4 \sqrt{x}(x+1) dx$

4. а) $\int_0^3 (3x^2 - 2x + 1) dx$

б) $\int_0^{\frac{\pi}{4}} \sin\left(2x + \frac{\pi}{3}\right) dx$

в) $\int_{-2}^{-1} \frac{5x^7 - 4x^6 + 2x}{x^3} dx$

5. а) $\int_{-2}^1 (2x^3 - 3) dx$

б) $\int_{\frac{\pi}{4}}^{\pi} \cos\left(3x - \frac{\pi}{3}\right) dx$

в) $\int_{-2}^{-1} \frac{3x^6 - 4x^5 - 7x^4 + 3x^2}{x^4} dx$

6. а) $\int_0^1 (2x^2 + 5) dx$

б) $\int_0^1 \frac{4}{(3x+2)^2} dx$

в) $\int_0^{\frac{3\pi}{4}} \left(1 - 2 \cos^2 \frac{x}{3}\right) dx$

7. а) $\int_0^3 (3x^2 - 3) dx$

б) $\int_1^2 \frac{3}{(2x-1)^2} dx$

в) $\int_{-\pi}^0 \left(2 \sin^2 \frac{x}{4} - 1\right) dx$

8. а) $\int_{-1}^3 (2x-1)(2x+1) dx$

б) $\int_2^7 \frac{4}{\sqrt{x+2}} dx$

в) $\int_0^{\frac{3\pi}{4}} 6 \sin \frac{x}{3} \cos \frac{x}{3} dx$

9. а) $\int_0^3 (x-1)(x+1) dx$

б) $\int_0^5 \frac{6}{\sqrt{3x+1}} dx$

в) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{7}{\cos^2 3x} dx$

10. а) $\int_{-1}^1 (6x^2 - 4x + 1) dx$

б) $\int_1^2 \frac{5x-2}{\sqrt[3]{x}} dx$

в) $\int_0^{\frac{\pi}{2}} \sin 2x \cos 3x dx$

11. а) $\int_{-2}^2 (12x^2 - 10x - 5) dx$

б) $\int_1^3 \frac{3x-1}{\sqrt{x}} dx$

в) $\int_0^{\frac{\pi}{3}} \cos 5x \cos 7x dx$

12. а) $\int_{-1}^2 (9x^2 - 2x + 1) dx$

б) $\int_{-2}^{-1} \frac{4}{x^2} \left(1 - \frac{2}{x}\right) dx$

в) $\int_{\frac{\pi}{4}}^{\pi} \cos^2 \frac{x}{2} dx$

13. а) $\int_{-1}^2 (3x^2 + 4x - 3) dx$

б) $\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$

в) $\int_{-\pi}^{\pi} \sin^2 3x dx$

14. а) $\int_{-1}^0 (2x-3)(2x+3) dx$

б) $\int_1^9 \left(2x - \frac{3}{\sqrt{x}}\right) dx$

в) $\int_1^5 \frac{dx}{\sqrt{2x-1}}$

15. а) $\int_0^1 (x-5)(x+5) dx$

б) $\int_0^{\pi} \sin x \cos x dx$

в) $\int_{-2}^{\frac{1}{3}} \frac{2dx}{\sqrt{10-3x}}$

16. а) $\int_{-1}^1 (x^2 + 1) dx$

б) $\int_0^3 \sqrt{x+1} dx$

в) $\int_1^2 \left(\frac{3}{x^2} + x^2 + 2\right) dx$

$$17. a) \int_0^1 (x-5)(x+5)dx$$

$$б) \int_2^6 \sqrt{2x-3}dx$$

$$в) \int_{-2}^{-1} \left(-\frac{5}{x^2} + x^4 - 3x \right) dx$$

$$18. a) \int_{-1}^0 (3x^2 + 1)dx$$

$$б) \int_0^3 \cos(4x-12)dx$$

$$в) \int_1^2 \frac{4x^5 - 3x^4 + x^3 - 1}{x^2} dx$$

$$19. a) \int_0^3 (3x^2 - 2x + 1)dx$$

$$б) \int_0^{\frac{\pi}{3}} \frac{1}{3} \sin\left(x - \frac{\pi}{3}\right) dx$$

$$в) \int_{-2}^{-1} \frac{5x^7 - 4x^6 + 2x}{x^3} dx$$

$$20. a) \int_{-1}^0 (2x-3)(2x+3)dx$$

$$б) \int_1^3 \sin(3x-6)dx$$

$$в) \int_3^6 \frac{dx}{2x-1}$$

$$21. a) \int_0^1 (2x^2 + 5)dx$$

$$б) \int_0^{\pi} \cos\left(2x - \frac{\pi}{3}\right) dx$$

$$в) \int_1^5 \frac{dx}{\sqrt{2x-1}}$$

$$22. a) \int_{-1}^0 (x+1)(x^2-2)dx$$

$$б) \int_0^{\frac{\pi}{4}} \frac{1}{2} \cos\left(x + \frac{\pi}{4}\right) dx$$

$$в) \int_{\frac{1}{3}}^3 \frac{dx}{\sqrt{10-3x}}$$

$$23. a) \int_{-1}^3 (2x-1)(2x+1)dx$$

$$б) \int_0^{\pi} \sin\left(\frac{x}{2} - \frac{\pi}{4}\right) dx$$

$$в) \int_2^3 \frac{6x^4 - 4x^3 + 7x^2 - 1}{x^2} dx$$

$$24. a) \int_0^3 (x-1)(x+1)dx$$

$$б) \int_1^8 4 \sqrt[3]{x} \left(1 - \frac{4}{x}\right) dx$$

$$в) \int_0^{\frac{\pi}{3}} \frac{5}{\sin^2\left(x + \frac{\pi}{3}\right)} dx$$

$$25. a) \int_{-1}^1 (6x^2 - 4x + 1)dx$$

$$б) \int_1^4 \sqrt{x} \left(3 - \frac{7}{x}\right) dx$$

$$в) \int_0^{\frac{2\pi}{3}} \sin\left(\frac{\pi}{3} - 3x\right) dx$$

$$26. a) \int_{-2}^2 (12x^2 - 10x - 5)dx$$

$$б) \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x) dx$$

$$в) \int_2^3 \frac{6x^4 - 4x^3 + 7x^2 - 1}{x^2} dx$$

$$27. a) \int_{-1}^2 (9x^2 - 2x + 1)dx$$

$$б) \int_{-3\pi}^0 \cos 3x dx$$

$$в) \int_{-1}^0 \sqrt[3]{1-2x} dx$$

$$28. a) \int_{-1}^2 (3x^2 + 4x - 3)dx$$

$$б) \int_{-2\pi}^{\pi} \sin 2x dx$$

$$в) \int_2^3 (5x-7)^{\frac{2}{3}} dx$$

2. Найти площадь фигуры, ограниченной линиями:

1. а) $y = (x-1)^2$; $y = 0$; $x = 2$.

б) $y = \frac{4}{x^2}$; $y = 4$; $x = 4$.

2. а) $y = 2x - x^2$; $y = 0$.

б) $y = \frac{1}{x^2}$; $y = 3$; $x = 4$.

3. а) $y = \frac{2}{x^2}$; $y = 0$; $x = 1$; $x = 4$.

б) $y = -x^2$; $y = -2$.

4. а) $y = \sqrt{x}$; $y = 0$; $x = 4$.

б) $y = x^3$; $y = 1$; $x = 2$.

5. а) $y = x^3$; $y = 0$; $x = 2$; $x = 4$.

б) $y = 1 - x^2$; $y = x^2 - 1$.

6. а) $y = x^2$; $y = 0$; $x = 3$; $x = 4$.

б) $y = 6x^2$; $y = (x-3)(x-4)$; $y = 0$.

7. а) $y = x^2 + 1$; $y = 0$; $x = -2$; $x = 1$.

б) $y = \sqrt{x}$; $y = x$.

8. а) $y = x^3 + 1$; $y = 0$; $x = 0$; $x = 2$.

б) $y = \sqrt{x}$; $y = x^2$.

9. а) $y = \sin x$; $x = \frac{\pi}{3}$; $x = \frac{2\pi}{3}$; $y = 0$.

б) $y = 1 + x^2$; $y = 3 - x$.

10. а) $y = \cos x$; $x = -\frac{\pi}{6}$; $x = 0$; $y = 0$.

б) $y = (2+x)^2$; $y = 2+x$.

11. а) $y = 4 - x^2$; $y = 0$.

б) $y = 9 - x^2$; $y = (x-2)^2 - 1$.

12. а) $y = -x^2 + 3x - 2$; $y = 0$.

б) $y = \sin x$; $y = \cos x$; $x = 0$; $x = \frac{\pi}{2}$.

13. а) $y = 1 - x^2$; $y = 0$.

б) $y = 6x - x^2$; $y = x + 4$.

14. а) $y = -x^2 + 4x - 3$; $y = 0$.

б) $y = 4 - x^2$; $y = x + 2$.

15. а) $y = \sqrt{x}$; $y = 0$; $x = 4$; $x = 9$.

б) $y = 2 - x^2$; $y = 1$.

16. а) $y = x^3 + 2$; $y = 0$; $x = 2$; $x = 0$.

б) $y = 1 - x^2$; $y = -x - 1$.

17. а) $y = -x^2 + 4x$; $y = 0$.

б) $y = x^2 - 3x + 2$; $y = x - 1$.

18. а) $y = -x^2 + 4$; $y = 0$.

б) $y = x^2 - 1$; $y = 2x + 2$.

19. а) $y = -x^3 + 1$; $y = 0$; $x = -2$; $x = 0$.

б) $y = \cos x$; $y = -x$; $x = 0$; $x = \frac{\pi}{2}$.

20. а) $y = \frac{1}{x^2}$; $y = 0$; $x = 1$; $x = 2$.

б) $y = \sin 2x$; $y = x - \frac{\pi}{2}$; $x = 0$.

21. а) $y = \frac{1}{\sqrt{x}}$; $y = 0$; $x = 1$; $x = 9$.

б) $y = \sin x$; $x = 0$; $x = \frac{\pi}{2}$; $y = -x$.

22. а) $y = \frac{1}{\sqrt{x}}$; $y = 0$; $x = 1$; $x = 4$.

б) $y = \cos \frac{x}{2}$; $y = x - \pi$; $x = 0$; $x = \pi$.

23. а) $y = \frac{1}{x^2}$; $y = 0$; $x = -1$; $x = -3$.

б) $y = -x^2 + 2x + 3$; $y = 3 - x$.

24. а) $y = \sin x$; $x = -\frac{\pi}{6}$; $x = \frac{\pi}{3}$; $y = 0$.

б) $y = x^3$; $y = 10 - x$; $x = 0$.

25. а) $y = \cos 2x$; $y = 0$; $x = -\frac{\pi}{6}$; $x = \frac{\pi}{3}$.

б) $y = x^2 - 4x$; $y = -(x-4)^2$.

26. а) $y = \cos x$; $x = -\frac{\pi}{4}$; $x = \frac{\pi}{4}$; $y = 0$.

б) $y = x^2 + 2x - 3$; $y = -x^2 + 2x + 5$.

27. а) $y = \sin \frac{x}{2}$; $y = 0$; $x = \frac{\pi}{2}$; $x = \pi$.

б) $y = x^2 - 6x + 9$; $y = (x+1)(3-x)$.

28. а) $y = 1 + \frac{1}{2} \cos x$; $y = 0$; $x = -\frac{\pi}{2}$; $x = \frac{\pi}{2}$.

б) $y = x^2 - 4x + 3$; $y = -x^2 + 6x - 5$.